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2006 J. Phys. A: Math. Gen. 39 4549

(http://iopscience.iop.org/0305-4470/39/17/S37)

INSTITUTE OF PHYSICS PUBLISHING JOURNAL OF PHYSICS A: MATHEMATICAL AND GENERAL

J. Phys. A: Math. Gen. **39** (2006) 4549–4553 doi:10.1088/0305-4470/39/17/S37

Magnetic dipolar interaction in two-dimensional complex plasmas

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Received 3 October 2005, in final form 8 February 2006
Published 7 April 2006
Online at stacks.iop.org/JPhysA/39/4549

Abstract

Various interactions can play a role between the mesoscopic dust grains of a complex plasma. We study a system composed of dust grains that have both an electric charge and a permanent magnetic dipole moment. It is assumed that the grains occupy lattice sites, as dictated by their Coulomb interaction. In addition, they possess a spin degree of freedom (orientation of magnetic dipole moment) that is not constrained by the Coulomb interaction, thus allowing for the possibility of equilibrium orientational ordering and 'wobbling' about the equilibrium orientations. As a result, collective modes develop. We identify in-plane and out-of-plane wobbling modes and discuss their dispersion characteristics both in the ferromagnetic and in the anti-ferromagnetic ground state.

PACS numbers: 52.27.Lw, 52.27.Gr, 52.35.Lv, 75.30.Ds

(Some figures in this article are in colour only in the electronic version)

1. Introduction

Historically, the problem of interacting dipoles that are fixed on lattice sites dates back several decades. Luttinger and Tisza published their seminal paper in 1946 [1], and since then investigations into one-, two- and three-dimensional arrays of dipoles have been extensive [2–4]. Studies of two-dimensional lattices have focused on the ground state configuration of the dipoles, with in-plane orientation [2, 5, 6], and its associated perturbations and excitations [3, 4, 7]. These latter works were directed towards understanding the properties of thin magnetic films (TMF). These systems consist of microscopic magnetic dipoles, the dynamics of which are well approximated by regarding the dipoles as two-dimensional classical vectors with an ability to rotate within the plane. The interaction includes the exchange interaction and the classical dipole–dipole potential. Recent collaborative experiments on complex dusty plasmas have led to the creation of two-dimensional (2D) lattices of mesoscopic grains [8, 9].

This has raised the possibility of creating and studying such lattices consisting of grains endowed with magnetic dipole moments, whose properties (both ground state configuration and collective excitations) may be accessible to direct experimental observation. In this paper we survey the possible ground state configurations and predict wave-like collective excitations, which we will refer to as wobbling modes. A preliminary study was given in [10]. These collective modes may have both in-plane and out-of-plane polarizations. The description of the ground state configurations is based on TMF investigations in the literature; the formalism for predicting the in-plane wobbling modes borrows heavily from the analysis of the spin wave excitations in TMF systems, although the physical interpretations of the processes are completely different. To the best of our knowledge, the out-of-plane wobbling modes have no analogue in earlier investigations. We trust that our predictions will motivate new experiments on this subject that in turn could lead to the discovery of new physical effects.

2. Magnetic dipoles in a complex plasma

We consider a strongly interacting 2D complex plasma in the crystalline state. We assume that the grains are ferromagnetic, i.e. they carry, in addition to their electric charge, a permanent magnetic dipole moment. Typical values of parameters for a complex plasma of ferromagnetic grains can be calculated from data given in [9, 11].

$$\mu \approx 10^{10}\mu_B\text{, magnetic dipole moment} \tag{1}$$

$$a \approx 10^{-4}\text{ m, lattice constant} \tag{2}$$

$$d \approx 10^{-6}\text{ m, particle diameter} \tag{3}$$

$$I \approx 10^{-26}\text{ kg m}^2\text{, moment of inertia} \tag{4}$$

$$Z \approx 10^3\text{–}10^4\text{, particle charge state} \tag{5}$$

$$m \approx 10^{-13}\text{ kg, particle mass.} \tag{6}$$

Based on these values, we estimate the ratio of the magnetic potential energy U_m to the electrostatic energy U_e to be

$$U_m/U_e = \frac{\mu^2}{a^3}\Bigg/\frac{Z^2e^2}{a} \approx 10^{-2}\text{–}10^{-4}. \tag{7}$$

These estimates indicate that the electrostatic interaction remains dominant, and in particular the electrostatic repulsion determines the lattice structure. Screening by the surrounding plasma particles leads to a Yukawa-type interaction, which then generates a 2D hexagonal crystal. As to a possible square lattice structure, the electrostatic monopole interaction by itself would not generate such a structure. Indeed, though 2D lattices of dust grains interacting via a screened Coulomb force have been created in several laboratory experiments (e.g. [12–14]), such a structure has never been observed. Nevertheless, whether the lattice structure does or does not remain unaffected by the magnetic interaction is an issue that awaits further clarification. Thus we consider both square and hexagonal lattices below.

3. Ground state

The ground state problem is similar to its TMF analogue. The Hamiltonian is that of a system of classical magnetic dipoles (S^α) interacting through the dipole–dipole potential $M^{\alpha\beta}(\mathbf{r})$

without exchange interaction:

$$V = -\frac{1}{2}\frac{\mu^2}{a^3}\sum_{i,j}\sum_{\alpha,\beta=x,y,z} M^{\alpha\beta}(\mathbf{r_i}-\mathbf{r_j})S_i^\alpha S_j^\beta \tag{8}$$

$$M^{\alpha\beta}(\mathbf{r}) = \frac{a^3}{r^3}\left(3\frac{r^\alpha r^\beta}{r^2} - \delta^{\alpha\beta}\right). \tag{9}$$

The ground state configuration for dipoles on a square lattice is well known to be anti-ferromagnetic (AF) (e.g. [7]). This ground state is continuously degenerate, with four neighbouring dipoles forming an AF vortical structure, where the vortical angle does not affect the ground state energy. The ground state configuration for dipoles on a hexagonal lattice is well known to be ferromagnetic, continuously degenerate with respect to the magnetization angle (e.g. [3]). However, both of these degeneracies are broken at finite—even very small—temperature [15, 16], because of the requirement of the minimization of the free energy. This leads to the AF columnar alignment as the minimum free-energy state for the square lattice, and the ferromagnetic next-nearest neighbour alignment for the hexagonal lattice.

4. Collective excitations

At finite temperature, the dipoles will oscillate (‘wobble’) around their equilibrium orientations, both in the plane and out of the plane. A coherent superposition of these oscillations results in collective wobbling mode excitations. In the Hamiltonian for the system the interaction part is the dipole–dipole interaction, while the kinetic term describes the oscillation energy of the dipole particles. In order to analyse the collective modes, we consider an equilibrium configuration with the dipoles aligned (either anti-ferromagnetically or ferromagnetically) along the x-axis. (In general the structure can be described in terms of sublattices; in the AF case a two-sublattice is used, with the unit cell consisting of two neighbouring particles, 1 and 2.) We then expand both the kinetic and potential energy terms to second order in the angular displacement, ψ^α, to generate a quadratic Hamiltonian. Adopting the in-plane and out-of-plane angles, ψ^y and ψ^z, respectively, as coordinates, and introducing the collective coordinates

$$\psi_{\mathbf{q}}^{A,\alpha} = \frac{1}{\sqrt{N}}\sum_i \psi^\alpha\, \mathrm{e}^{\mathrm{i}\mathbf{q}\cdot\mathbf{r}_i^A} \tag{10}$$

(the index $A = 1, 2$ enumerating sublattice positions within the unit cell) we obtain

$$H = \frac{1}{2}I\sum_{\mathbf{q}}\dot{\psi}_{\mathbf{q}}^{A,\alpha}\dot{\psi}_{-\mathbf{q}}^{A,\alpha} + \frac{1}{2}\frac{\mu^2}{a^3}\sum_{\mathbf{q}}\psi_{\mathbf{q}}^{A,\alpha}F^{AB,\alpha\beta}(\mathbf{q})\psi_{-\mathbf{q}}^{B,\beta}. \tag{11}$$

The elements of the $\mathbf{F}$ matrix are expressible in terms of the elements of the primitive dynamical matrix $\mathbf{M}(\mathbf{q})$ for a 2D Coulomb lattice $M^{\alpha\beta}(\mathbf{q}) = \sum_i M^{\alpha\beta}(\mathbf{r}_i)\,\mathrm{e}^{\mathrm{i}\mathbf{q}\cdot\mathbf{r}_i}$ [17]. The eigenvalues of the $\mathbf{F}$ matrix, $\lambda(\mathbf{q})$, are then easily worked out, and they give the frequencies of the collective modes of the system:

$$\omega(\mathbf{q}) = \left[\frac{\mu^2}{Ia^3}\lambda(\mathbf{q})\right]^{\frac{1}{2}}. \tag{12}$$

For the AF square lattice the two sublattices generate four modes: in-plane, in-phase (+, in); in-plane, out-of-phase (−, in); out-of-plane, in-phase (+, out); and out-of-plane, out-of-phase (−, out).

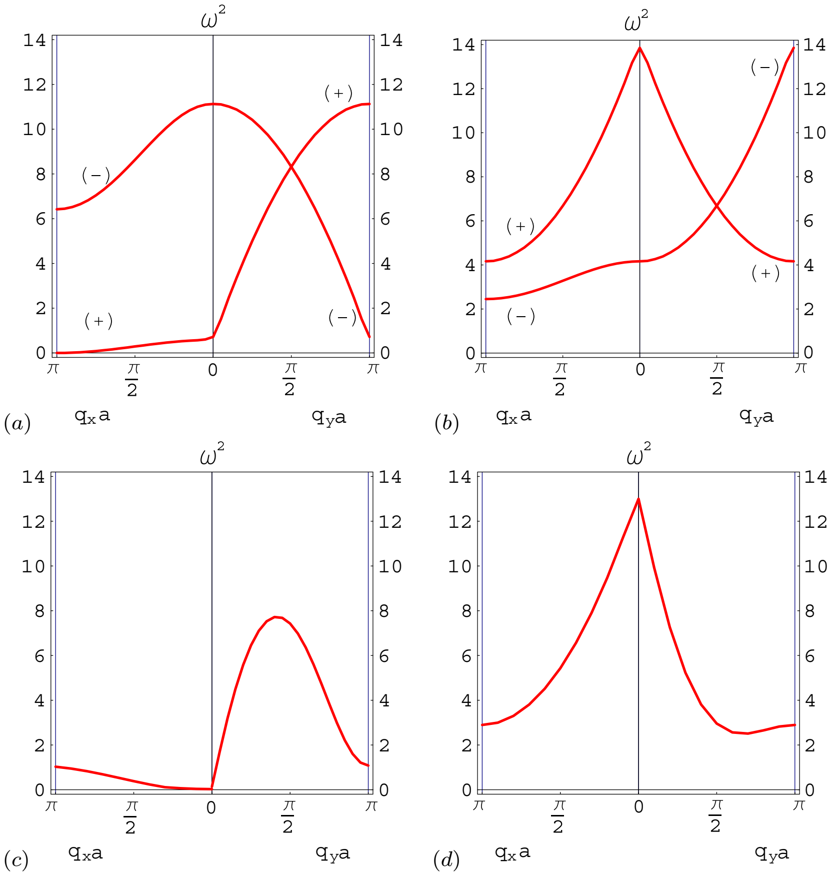

Figure 1. Dispersions for the wobbling modes (in units of $\frac{\mu^2}{Ia^3}$) for (*a*) square lattice in-plane ; (*b*) square lattice out-of-plane; (*c*) hexagonal lattice in-plane; (*d*) hexagonal lattice out-of-plane. $n_{\text{square}} = n_{\text{hexagonal}}$, thus $a_{\text{square}} \neq a_{\text{hexagonal}}$.

$$\lambda_{\text{in}}^{+}(\mathbf{q}) = M_{11}^{xx}(0) - M_{12}^{xx}(0) - \left(M_{11}^{yy}(\mathbf{q}) + M_{12}^{yy}(\mathbf{q})\right) \tag{13}$$
$$\lambda_{\text{in}}^{-}(\mathbf{q}) = M_{11}^{xx}(0) - M_{12}^{xx}(0) - \left(M_{11}^{yy}(\mathbf{q}) - M_{12}^{yy}(\mathbf{q})\right) \tag{14}$$
$$\lambda_{\text{out}}^{+}(\mathbf{q}) = M_{11}^{xx}(0) - M_{12}^{xx}(0) - \left(M_{11}^{zz}(\mathbf{q}) + M_{12}^{zz}(\mathbf{q})\right) \tag{15}$$
$$\lambda_{\text{out}}^{-}(\mathbf{q}) = M_{11}^{xx}(0) - M_{12}^{xx}(0) - \left(M_{11}^{zz}(\mathbf{q}) - M_{12}^{zz}(\mathbf{q})\right). \tag{16}$$

For the ferromagnetic hexagonal lattice there is no sub-lattice and only the in-phase mode exists.

$$\lambda_{\text{in}}^{+}(\mathbf{q}) = M^{xx}(0) - M^{yy}(\mathbf{q}) \tag{17}$$

$$\lambda_{\text{out}}^{+}(\mathbf{q}) = M^{xx}(0) - M^{zz}(\mathbf{q}). \tag{18}$$

5. Wobbling modes

The dispersions of the wobbling modes are shown in figure 1, both for the square and hexagonal lattices. Frequencies are given in units of $\frac{\mu^2}{Ia^3}$. Note that the ferromagnetic in-plane mode is a Goldstone mode: $\omega \to 0$ for $q \to 0$. In the AF case, the in-plane, in-phase mode behaves as a quasi-Goldstone mode: $\omega \to 0$ for $q_x a \to \pi, q_y \to 0$. The existence of Goldstone modes is the consequence of the continuous degeneracies of the ground states [18]. The out-of-plane modes are always gapped modes. The in-plane modes are in one-to-one formal correspondence with the low-lying spin wave excitations of the TMF systems [3, 4, 7], although they obviously represent a completely different physical phenomenon, since the dipoles are associated with a massive object with a moment of inertia. For this reason, the out-of-plane modes do not appear to have an analogue in other physical systems such as TMF.

6. Summary and discussion

In this paper, we have established the existence of novel in-plane and out-of-plane wobbling modes in a 2D lattice of mesoscopic grains endowed with a permanent magnetic dipole moment, in addition to an electric charge. Since the grain temperature is of the order of room temperature, the wobbling modes would be expected to be thermally excited. Interaction with the surrounding plasma and neutral gas may further excite these wobbling modes. We hope that our findings will stimulate related experimental efforts in laboratory complex plasmas. There are intriguing issues associated with the experimental realization of the system studied in this paper, such as the effect of non-electrostatic forces or thermal motion on the wobbling of the grains, as well as how to diagnose these wobbling modes.

Acknowledgments

This work has been partially supported by NSF grant no. PHYS-0206695, DOE grant no. DE-FG02-03ER54716, DOE grant no. DE-FG02-04ER54804. The authors thank Michael Giersig for discussions.

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